University of California, Berkeley **Economics 230a**
Department of Economics **Fall 2017**

Problem Set #1

(due 10/17/17)

1. Consider an economy in which relative producer prices are fixed and a representative household, with a unit endowment of labor, maximizes the following utility function:

$$U(c_1, c_2, l) = (c_1 - a_1)^{\beta_1}(c_2 - a_2)^{\beta_2} l^{1-\beta_1-\beta_2}$$

(where c_1 and c_2 are consumption goods and l is leisure), subject to the budget constraint:

$$p_1 c_1 + p_2 c_2 + wl = w$$

A. Derive an explicit solution (i.e., in terms of prices and preference terms a_i and β_i) for the excess burden of taxes on c_1, c_2, and l as a function of the original, undistorted prices of the three goods (p_1^0, p_2^0, and w^0), the distorted prices (p_1^1, p_2^1, and w^1) and a fixed utility level.

B. Show that excess burden equals zero if $p_i^1 = (1+\theta)p_i^0$, i = 1, 2, and $w^1 = (1+\theta)w^0$ for some constant θ.

C. Compare the values of excess burden based on utility levels achieved in the absence and in the presence of taxation, $V(p_1^0, p_2^0, w^0)$ and $V(p_1^1, p_2^1, w^1)$.

D. Using the measure derived in part A, show that the marginal excess burden for an increase in a tax *or* subsidy on good 2 is positive. (*Hint*: relate the change in excess burden to the sign of $(p_2^1 - p_2^0)$.)

2. Consider a model of household production, in which the representative household maximizes the utility of market goods *X* and home goods *Z*, *U*(*X*,*Z*). The household has one source of income, labor, and derives no utility directly from leisure. It supplies some of its unit labor endowment to the market and uses the rest in home production of *Z*. Labor supplied to the market goes into the production either of *X* or an intermediate good, hired day care services, *M*. *X* and *M* are each produced in the market subject to constant returns to scale using labor, and the household produces *Z* subject to constant returns to scale using two factors of production: the labor it withholds from the market, *h*, and hired day care services, *M*.

A. Write down the household's utility and budget constraint (without taxes) as functions of *X*, *M*, and *h*, and show that the problem is equivalent to one in which *X* and *M* are consumption goods with fixed producer prices and *h* is leisure.

B. Suppose that the government has imposed a proportional tax on labor income to raise revenue. It is suggested that efficiency might be enhanced by adding a subsidy to market day care services, *M*, in order to encourage individuals to work. Show that this will be true if and only if the cross-elasticity of demand for *X* with respect to *M*, ε_{XM}, is lower than the cross-elasticity of demand for home labor, *h*, with respect to *M*, ε_{hM}. (*Hint*:

consider the equivalent tax scheme involving taxes on X and M and derive a condition for these taxes to be equal, using the properties of the utility function derived in part A.)

3. In the Harberger two-sector model, labor bears 100% of an excise tax on sector-X output if the ratio of capital income to gross income (including the excise tax) is unchanged.

   A. For the same assumptions as in the standard Harberger model (e.g., fixed overall supplies of labor and capital, no initial distortions), show that this outcome requires that sector X be more labor intensive than sector Y.

   B. Using expressions from Lecture Note 6, derive a condition that depends only on factor shares (θ), factor allocations (λ) and elasticities of substitution (σ) for labor to bear at least 100% of an excise tax in sector X.

   C. Assume that sector X is more labor intensive than sector Y, so that (from the result in part A) it is possible for labor to bear 100% of an excise tax on sector X. Using the expression you derived in part B, show that, in the limit as goods X and Y become perfect substitutes in consumption (i.e., as $\sigma_D \to \infty$), labor must bear at least 100% of the tax.

4. Consider an economy with overlapping generations, each with a single agent who lives for two periods. (Let generation t be the generation that is young in period t.) The world interest rate is fixed at r. The timing convention is that government debt is issued at the beginning of the period, and taxes, transfer payments and government purchases occur at the end of the period. Initially, in year t, the government has no national debt outstanding, and operates a social security system that transfers 1 unit of output to the older individual from the younger individual in each period.

   A. Following the expression in Lecture Note 7, write down the government's intertemporal budget constraint (GIBC) in year t in terms of national debt, government purchases and government net taxes (taxes less transfers), and show that the government's policy satisfies the GIBC.

   B. Now write down the GIBC in its alternative formulation, in terms of the initial level of debt, government purchases, and the generational accounts for all existing and future generations. Solve for the generational account for each generation, and show that this version of the GIBC is also satisfied under current government policy.

   C. Suppose that, at the end of the current period, t (i.e., at the beginning of period t+1), the government eliminates the social security system by issuing bonds to pay for the current elderly agent's benefit. Assuming that government services the debt using equal taxes on each future elderly generation, solve for the tax needed to satisfy the GIBC. Solve for all generational accounts after this policy change, and show that they are the same as for the original social security system in part B.